Calculation of descriptive statistics by devices with low computational resources for use in calibration of V2I system

Marzena Banach[1,2], Katarzyna Kubiak[1,3] and Rafał Długosz[1,4]
[1] Aptiv Services Poland
ul. Podgórki Tynieckie 2, 30-399, Kraków, Poland
[2] Poznan University of Technology, Institute of Architecture and Spatial Planning,
Nieszawska 13C, 61-021 Poznan, Poland
E-mail: marzena.banach@erba.com
[3] Adam Mickiewicz University
Faculty of Mathematics and Computer Science
ul. Umultowska 87, 61-614 Poznan, Poland
[4] UTP University of Science and Technology
Faculty of Telecommunication, Computer Science and Electrical Engineering
ul. Kaliskiego 7, 85-796, Bydgoszcz, Poland
E-mail: rafal.dlugosz@aptiv.com, rafal.dlugosz@gmail.com

***Abstract*—In this work we present modified iterative methods for computing basic statistical quantities (mean, variance) for use in the calibration process of a system based on V2I (vehicle-to-infrastructure) communication devices. Such devices, mounted in the road and urban infrastructure (RSU – road side equipment) may be used as support for autonomous vehicles moving in urban environment. Calibration is necessary to determine the positions of the RSUs in global coordinate system (GCS) and to record this information in their internal memory. The proposed modifications to conventional iterative algorithms aim at adapting these methods to the application in devices with low computational abilities or directly at the transistor level in specialized integrated circuits.**

***Keywords*—Intelligent transportation system, V2I communication, intelligent urban infrastructure, statistical methods, low power devices**

I. Introduction

In the design of advanced driver assistant systems (ADAS) and ultimately autonomous driving, the communication between vehicles (V2V – vehicle-to-vehicle) and between vehicles and infrastructure (V2I – vehicle-to-infrastructure) is commonly assumed. In the case of the V2I system, the devices communicating with vehicles will be permanently located in the elements of the urban and road infrastructure. The system based on the V2I communication can perform various tasks. In the simplest approach it may inform vehicles about various unusual traffic situations which may affect driving safety. In the future, the V2I system may also be used to support autonomous driving of vehicles. Appropriately distributed dense network of devices installed in the road infrastructure (RSU) may become a frame of reference for moving vehicles.

In every situation mentioned above, one of important tasks is to determine, with appropriate precision, the positions of the RSU devices relative to moving vehicles. The passing vehicle must be able to distinguish whether it is the addressee of the information it receives through the V2I system (e.g. from an intelligent traffic sign). The required locating accuracy will depend on how much this information is critical to the safety of the moving vehicle. High precision will be important especially in the situation of a direct support for autonomous driving.

Various methods for determining the positions of the RSU may be considered, including communication methods used in real time locating systems (RTLS). The GPS may provide data even with large random error, especially in a dense urban area. For this reason, statistical techniques may be necessary to enhance the positioning of the devices mounted in the urban infrastructure. In our work we focus on mathematical/statistical methods for calibrating the V2I system using the RTLS, assuming that the GPS is not used. An idea of such a calibration approach has been recently reported by us in a patent application [1].

The aim of this work is to contribute to simplifying the hardware structure of the RSU devices in order to reduce the energy consumed by these devices and to enable, to a higher extent, their operation with energy scavenged from the environment. It is important as the access to solar energy, for example, may be limited in dense urban areas.

In this paper we present modified iterative methods for computing selected basic statistical quantities, i.e. mean and variance. However, the proposed methods may be easily adapted to the calculations of central moments of higher orders. In the comparison with conventional iterative methods, reported in the literature, we use only simple arithmetical operations such as summing, subtracting, multiplying and shifting the bits. In this approach, the methods may be easily implemented in hardware, including digital and analog application-specific integrated circuits (ASIC). This allows to simplify the hardware structure of the RSU devices. The resultant low energy consumption and small sizes of the devices will positively impact their price. Taking this into account, it can be said that this work is part of the emerging trend of implementing various ADAS functions in hardware [2], [3]. An important part of this trend is also a hardware implementation of fast regulation systems [4].

The paper is organized as follows. The next section provides a detailed background and motivation behind undertaking the proposed research topic. In the following section, the proposed method for computing mean and variance is described in detail. Then, selected simulation results are presented in sections IV and V. The conclusions are formulated in the last section.

II. State-of-the art study / prior art

The literature provides a number of solutions that anticipate the application of the V2I system in the ADAS functions and as a support for autonomous driving of vehicles. Support may consist of providing warning messages to passing vehicles about dangerous situation on the road [5], [6]. Such situations include, but are not limited to, the possibility of a collision with a pedestrian or other vehicle, an accident ahead. Messages can also be used to optimize the vehicle route in the city. This topic is also related to works which aim at standardizing the methodology of the communication (protocols, standards regarding formats of the V2I messages, etc.).

One of the important aspects discussed in the literature is to ensure the positioning of RSU devices with the appropriate precision [7]–[10]. These methods are usually based on a direct use of the GPS devices [7]–[9]. GPS-based solutions have their limitations. One of the important issues is the power dissipation of such devices (access to power line may be required). This, in turn, will impact the ease of assembly of such devices in the elements of the urban infrastructure. In the case of the systems not directly related to driving safety, the power supply may be based on natural energy sources (solar, wind). However, this is not recommended in case of the systems directly interfering with the maneuvers performed by vehicles.

Another method of positioning the RSU devices is a direct communication between these devices and passing vehicles [9], [10]. For this purpose, solutions which base on the IR-UWB (impulse radio – ultra wide band) technology and are known from the RTLSs may be used. This technology offers a high positioning accuracy [11], [12], [13] and is therefore proposed as one of the possibilities of supporting the V2I system. In such systems, one of the devices sends a signal received by the second device which, in turn, sends a feedback signal received by the first device. The position of the device is calculated on the basis of the measurement of the elapsed time between sending and receiving the signal. It is also important to know the trajectory of the moving device. While the measurements are repeated, triangulation methods may be used to determine the position of the device [12].

The trajectory of the vehicle (local and in the GCS) may be determined basing on the vehicle's GPS, as well as data provided by on-board sensors (yaw rate, velocity, etc.). Theoretically, on the basis of subsequent measurements of the distance to a given RSU, its location in the GCS can be determined. In practice, however, various factors may affect the accuracy of the measurements and computations (external temperature, noise). The response time of the device itself may, for example, depend on the temperature, translating into a positioning error [14].

Another factor affecting the accuracy of determining the position of the RSU in respect to the GCS, is the positioning accuracy offered by the GPS of the moving vehicle. Another unfavorable factor is the limited accuracy of the on-board sensors, on the basis of which the trajectory of the vehicle is determined. Data provided by these sensors may be noisy and require filtering that, in turn, introduces delays that depend on the filter parameters [15]. All of these factors can cause that even with many distance measurements between a given vehicle-RSU pair, instead of one position, a group of apparent positions will appear [14]. Taking all of the described reasons into consideration, the overall V2I system may require a calibration basing on statistical methods. The specificity of the system causes that low computation complexity methods need to be developed.

III. The proposed methods for computing statistical quantities

Variance is one of the basic statistical quantities computed for various purposes. The standard (theoretical) method is given as follows:

$$\mathrm{Var}(X) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \mu)^2 \tag{1}$$

To use this method, the overall dataset X has to be known in advance. In the proposed application, however, we assume that data samples used to calculate statistical quantities will we provided to the RSU device at a constant rate. For this reason, it is not advised to save all data samples in its memory. In the iterative approach, only the instantaneous values of the calculated mean and variance have to be stored. They are updated for each new data sample. The values of the computed quantities may exhibit a variability over time, which will depend on environmental conditions. For this reason, in one of the solutions presented below, we assume the use of a window in terms of time (a time window) or in terms of a given amount of samples (a sample window), which allows for a drift of computed values – adaptation to varying external conditions.

A. *Iterative computation of mean and variance*

Initially, the variables representing the mean and the variance are set as follows:

$$\mathrm{Mean}_1 = x_1 \tag{2}$$

$$\mathrm{Var}_1 = 0 \tag{3}$$

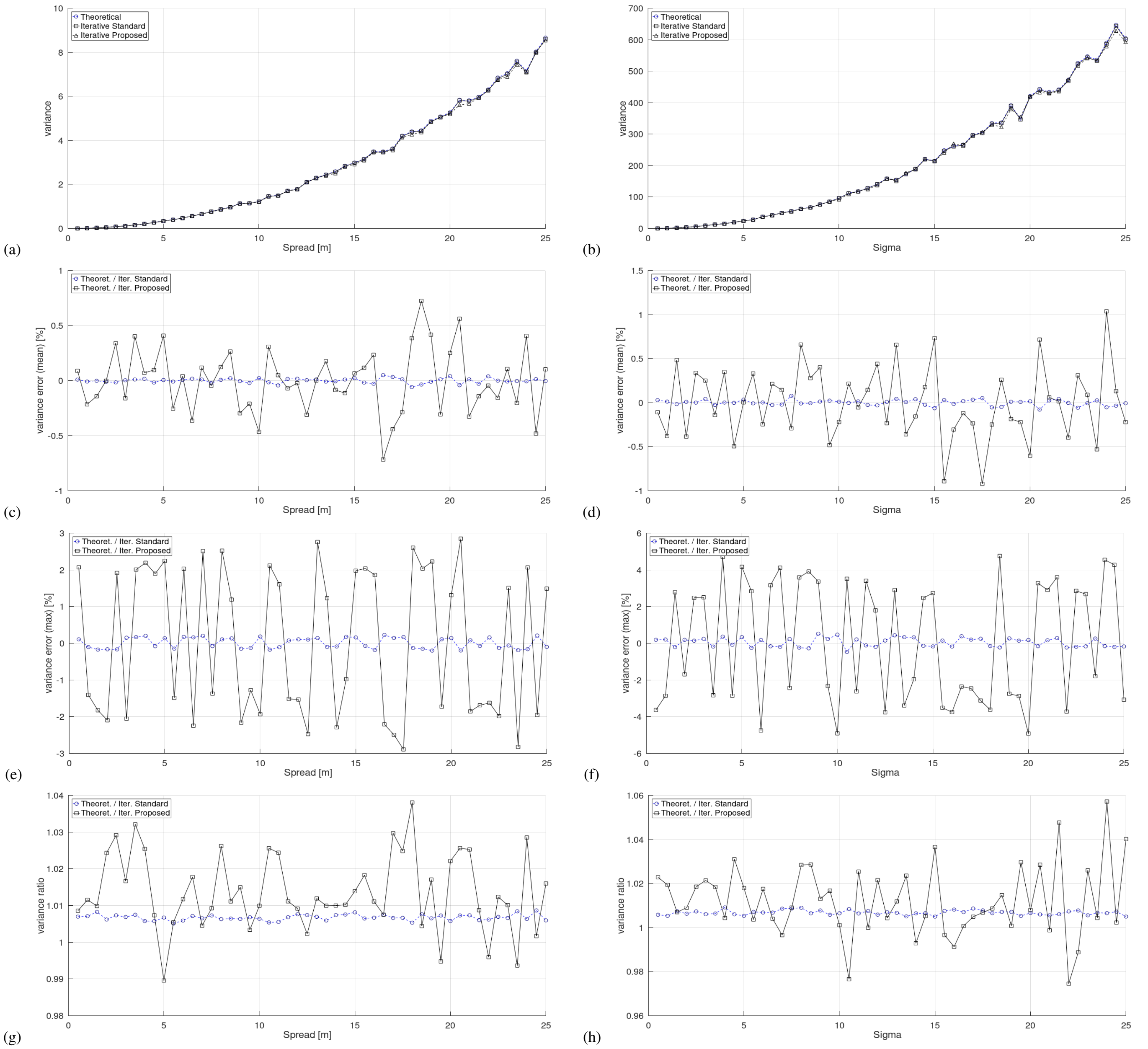

Fig. 1. Comparison of various methods used for calculation of variance for an exemplary case of 1000 samples, for uniform (left) and normal (right) distributions:
(a, b) Variance values for different parameters of the input signal (spread, sigma),
(c, d) averaged errors from 10 computation series for particular signal parameters,
(e, f) maximum errors for 10 series for particular signal parameters,
(g, h) ratio between theoretical and iterative methods for a single series.

where x_1 is the first recorded sample.

For the subsequent samples, i, the updates of both the quantities are calculated as follows [16]:

$$\mathrm{Mean}_i = [\mathrm{Mean}_{i-1} \cdot (i-1) + x_i]/i \tag{4}$$

$$\mathrm{Var}_i = [\mathrm{Var}_{i-1} \cdot (i-1) + (x_i - \mathrm{Mean}_i)^2]/i \tag{5}$$

Formulas 4 and 5 may be rewritten as:

$$\mathrm{Mean}_i = \mathrm{Mean}_{i-1} + [x_i - \mathrm{Mean}_{i-1}]/i \tag{6}$$

$$\mathrm{Var}_i = \mathrm{Var}_{i-1} + [(x_i - \mathrm{Mean}_{i-1})^2 - \mathrm{Var}_{i-1}]/i \tag{7}$$

Formulas 6 and 7 are consistent with the work published by West in 1979 [16]. They contain the division operation by the factor i that is equal to the number of the input samples already processed (iteration). To make this approach more suitable for the use in the devices with low computational power or at the transistor level, we propose the following modification of 6 and 7:

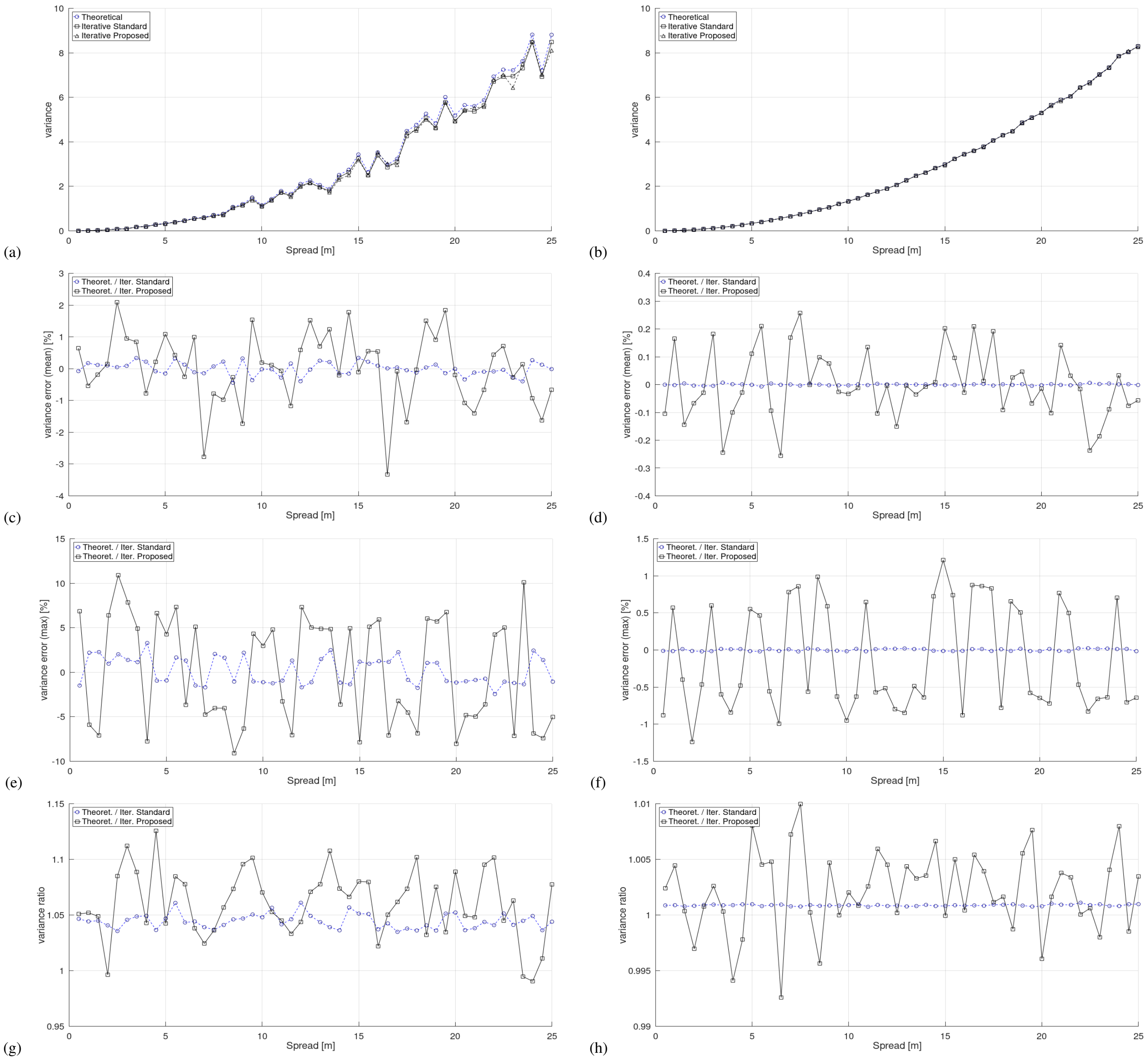

Fig. 2. Selected results as in Fig. 1, for samples from a uniform distribution in the input dataset, for 100 samples (left) and 10000 samples (right).

$$\text{Mean}_i = \text{Mean}_{i-1} + [x_i - \text{Mean}_{i-1}]/c \quad (8)$$

$$\text{Var}_i = \text{Var}_{i-1} + [(x_i - \text{Mean}_{i-1})^2 - \text{Var}_{i-1}]/c \quad (9)$$

The difference is the replacement of the index i in the denominators of both equations with a new variable c. This variable is initially set to 2. In the subsequent iterations its values are determined according to the following scheme:

$$\textbf{if } (i > c)\{c = c * 2;\} \quad (10)$$

As a result, the c variable is always a power of 2:

$$c \in \{2, 4, 8, 16, 32, \dots\} \quad (11)$$

The use of the c variable, defined as above, enables the substitution of the division operation with an operation of bits shifting in 10. This operation is substantially simpler than the typical division. In the case of the fixed point numbers, the division by 2^k is accomplished by shifting the bits to the right by k positions.

The expressions in numerators of 8 and 9 may be either positive or negative. For this reason, while shifting the bits, the k most significant bits have to be filled with an appropriate value. In case of two's complement code, the most significant bit (MSB) is equal to '0' for positive values and to '1' for negative ones. The value of this bit may be directly used as the filling signal. For example, dividing an 8-bit number 97 (binary 01100001) by 4 we get 24 (B: 00011000) – three

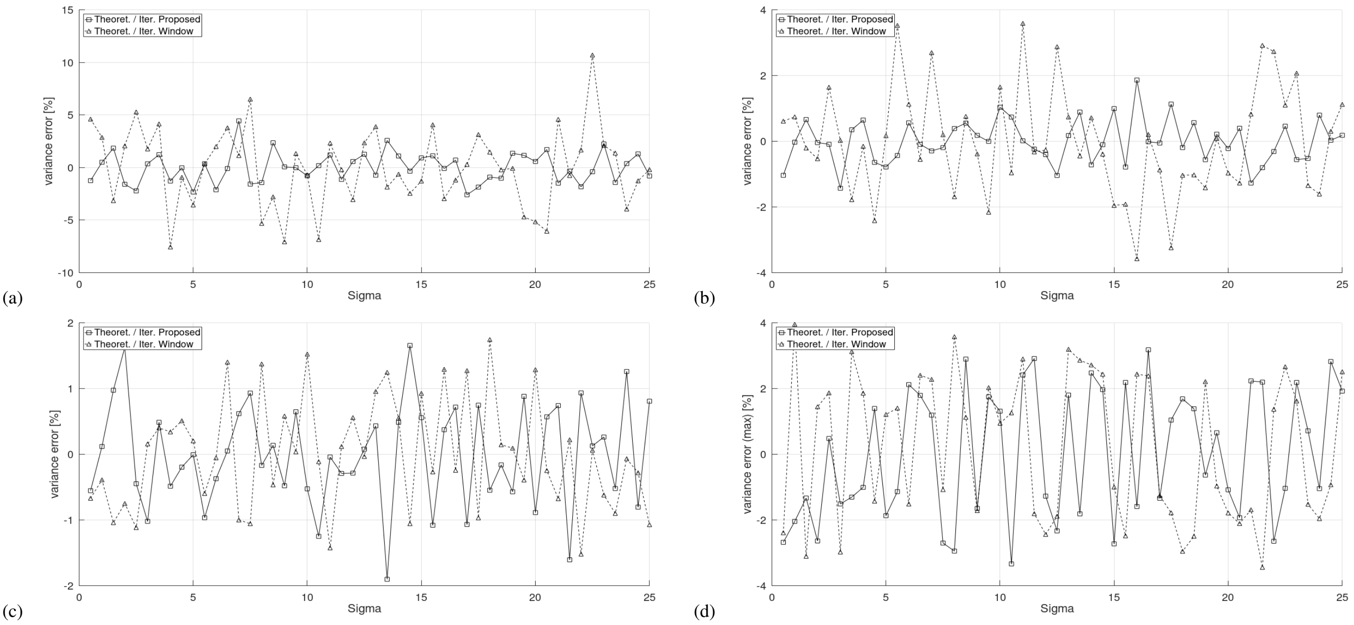

Fig. 3. Selected results illustrating the impact of windows limiting the number of samples on the variance error:
(a) dataset with 1000 samples and the length of the window set to 128, for 1 series,
(b) dataset with 10000 samples and the length of the window set to 1024, for 1 series,
(c) dataset with 10000 samples and the length of the window set to 2048, for 1 series,
(d) dataset with 2000 samples and the length of the window set to 512 – maximum error for 5 series.

MSBs are filled with '0'. On the other hand, for -97 (B: 10011111) divided by 4 we get -25 (B: 11100111) – three MSBs are filled with '1'.

Finally, formulas 8 and 9 may be expressed as follows:

$$\mathrm{Mean}_i = \mathrm{Mean}_{i-1} + [x_i - \mathrm{Mean}_{i-1}] \gg k \tag{12}$$

and

$$\mathrm{Var}_i = \mathrm{Var}_{i-1} + [(x_i - \mathrm{Mean}_{i-1})^2 - \mathrm{Var}_{i-1}] \gg k \tag{13}$$

where '»' is the bit shift operation by k positions to the right. The k variable can be determined according to a scheme shown below for each iteration. Initially, both involved variables are set as follows: $c = 2$, $k = 1$.

$$\textbf{if } (i > c)\{c = c \ll 1; k = k + 1;\} \tag{14}$$

B. Using windows

One of the possibilities is freezing the values of the c and k variables after reaching a given number of samples. Let us denote it as Thr. The Thr may be equal, for example, to the number of samples required to calibrate the RSU device. In this approach, formula 14 may be rewritten as follows:

$$\textbf{if } ((i > c) \ \&\& \ (i < \mathrm{Thr}))\{c = c \ll 1; k = k + 1;\} \tag{15}$$

IV. Investigation of the Results

This section presents selected simulation results whose aim was to compare the proposed methods with the state-of-the art methods. In order to carry out the tests, an application enabling comprehensive comparative analysis was created in the Octave environment. The test were performed for different parameters of the input signal. The samples of the input signal were drawn randomly from two distributions: normal and uniform. The parameters sigma and spread were changed in the range from 0.5 to 25, with the step of 0.5 (producing 50 test cases).

In the paper two groups of tests were presented. In the first group, the calculations were repeated 10 times for the spread and sigma parameters, each time for a newly drawn input signal sample. Consequently, 500 different signals were analyzed in a single test cycle. Then, the errors introduced by the proposed methods ware calculated relative to the theoretical values given by 1. As a reference point, the general mean of variance from the 500 tests for a given cycle for individual iterative methods was assumed separately. Next, the mean value of the error as well as the maximum value of the error (i.e. the worst case) was calculated relative to the general mean for particular parameters of the input signal.

Exemplary results of the first group of tests are shown in Figs. 1 and 2. Fig. 1 presents the results for 1000 signal samples from the uniform and normal signal distributions. Fig. 2 shows the results for the the signal from a uniform distribution for 100 and 10,000 input samples, respectively. Plots (a) and (b) in both Figures show absolute values of variance for the theoretical, standard iterative and proposed iterative methods. Then, plots (c) and (d) present mean deviation of both iterative methods from the theoretical method. Finally, plots (e) and (f) illustrate the ratio of the results of both iterative methods to the results of the theoretical method for a single series.

The second group of tests examined the effect of introducing a window limiting the number of signal samples. As a result, variance is calculated basing on a filter with a shift register. Exemplary results are shown in Fig. 3 for different numbers of input signal samples and for different window lengths for the proposed methods. Plots (a), (b) and (c) present selected results for single series, and plot (d) shows the maximum values of error for five series.

If a drift of values of statistical parameters is observed, then using a window may be of importance as it causes the proposed method to work as a filter.

V. Discussion of results

It is worth to notice that both iterative methods introduce a gain, which is bigger than 1, as shown in plots (g) and (h) in Figs. 1 and 2. The gain is relatively constant for particular parameters of the input signal, which change over a wide range, and is equal to about 1.05. This possibly enables its correction.

Both iterative methods display deviations from the theoretical method, which become smaller as the number of samples increases. In the case of 100 samples the errors are of 3 and 11 % for the standard and the proposed iterative methods, respectively (see Fig. 2 (e)). In the case of 1000 samples, in the proposed iterative method the maximum error decreases to 3 % and 5 % for the uniform and the normal distributions, respectively (Fig. 1 (e, f)). Furthermore, it decreases to 1.5 % for 10000 samples (Fig. 2 (f)). However, the error values of even 5 % can be omitted in many applications.

As expected, the introduction of a window reduces the decrease of the error with the increasing absolute number of the samples. When the ratio of the absolute number of samples to the length of the window is greater than 4 – 5, then the errors are comparable in both cases, as can be particularly seen in Fig. 3 (d) (maximum values for five series).

VI. Conclusions

In this work we presented modified iterative methods for computing selected statistical quantities, i.e. mean and variance. However, the proposed formulas can be easily adapted for calculation of central moments of higher orders, which are used to determine the values of kurtosis and skewness of a probability distribution.

The proposed methods eliminate the computationally expensive division operation, which makes them very easy to implement in hardware or in systems with limited computing resources.

Comprehensive tests of the proposed modified methods for various parameters were performed. Errors introduced by the proposed methods do not exceed 1.5 to 10 % (depending on the number of samples), which in many situations may be neglected. The presented results are only a few examples and were chosen from more than 100,000 tests performed in general.

References

[1] Banach M., Długosz R., "Method to improve the determination of a position of a roadside unit, roadside unit and system to provide position information", Application number: 18200799.7, Date of filing: 16-Oct-2018, Applicant: Aptiv Technologies Limited, European Patent Office

[2] Komorkiewicz, M., Kryjak, T., Chuchacz-Kowalczyk, K., Skruch, P., Gorgoń, M., "FPGA based system for real-time structure from motion computation," *Proceedings of the DASIP 2015 Conference on Design & Architectures for Signal & Image Processing*, 23-25.09.2015, Kraków, Poland.

[3] Komorkiewicz, M., Turek, K., Skruch, P., Kryjak, T., Gorgoń, M., "FPGA-based hardware-in-the-loop environment using video injection concept for camera-based systems in automotive applications", *Proc. of the Conference on Design & Architectures for Signal & Image Processing* (DASIP), Book Series: J.F. Nezan (Ed.) *Conference on Design and Architectures for Signal and Image Processing*, pp. 183-190, 2016, 12-14.10.2016, Rennes, France.

[4] Skruch, P., Dlugosz, M., Mitkowski, W., "Mathematical methods for verification of microprocessor-based PID controllers for improving their reliability," *Eksploatacja i Niezawodnosc – Maintenance and Reliability*, vol. 17, no. 2, pp. 327-333, 2015, http://dx.doi.org/10.17531/ein.2015.3.1.

[5] Vadim A. Butakov, Petros Ioannou, "Personalized Driver Assistance for Signalized Intersections Using V2I Communication", IEEE Transactions on Intelligent Transportation Systems, Vol: 17, Issue: 7, July 2016, pp.1910-1919

[6] Olaverri-Monreal C., Errea-Moreno J., Díaz-Álvarez A., "Implementation and Evaluation of a Traffic Light Assistance System Based on V2I Communication in a Simulation Framework", Journal of Advanced Transportation, Vol. 2018, Article ID 3785957, 11 pages, https://doi.org/10.1155/2018/3785957

[7] Kim Tae Won, Jung Jae Il, "GPS UWB V2x Autonomous Driving Method And System for Determining Position of Car Graft on GPS UWB And V2X," KR20170071207 (A) – 2017-06-23

[8] Jian Wang, Yang Gao, Zengke Li, Xiaolin Meng, Craig M. Hancock, "A Tightly-Coupled GPS/INS/UWB Cooperative Positioning Sensors System Supported by V2I Communication", Sensors, 16(7), 944; doi:10.3390/s16070944, 2016.

[9] Amini A., Vaghefi R.M., de la Garza M., Michael Buehrer R.M., "Improving GPS-Based Vehicle Positioning for Intelligent Transportation Systems", IEEE Intelligent Vehicles Symposium (IV), Electronic ISBN: 978-1-4799-3638-0, DOI: 10.1109/IVS.2014.6856592, Jul. 2014

[10] Fascista A., Ciccarese G., Coluccia A., Ricci G., "A Localization Algorithm Based on V2I Communications and AOA Estimation", IEEE Signal Processing Letters, Vol. 24, Issue: 1, Jan. 2017

[11] Martynenko D., Klymenko O., Fischer G., Kreiser D., "Implementation and Evaluation of a Two-Way-Ranging Module based on an IR-UWB Chip-Set", 12th Workshop on Positioning, Navigation and Communication (WPNC'15), Niemcy, 2015

[12] Hassan O., Adly I., Shehata K.A., "Vehicle Localization System based on IR-UWB for V2I Applications", 8th International Conference on Computer Engineering & Systems (ICCES), 26-28 Nov. 2013

[13] Llorente R., Morant M., "UWB technology for safety-oriented vehicular communications", Proc. SPIE 9807, Optical Technologies for Telecommunications 2015, 980705; doi:10.1117/12.2234430, 26 marca 2016.

[14] Banach M., Długosz R., "Real-time Locating Systems for Smart City and Intelligent Transportation Applications", 30th International Conference on Microelectronics (MIEL), Nis, Serbia, Oct. 2017, pp. 231–234.

[15] Dlugosz, R., Szulc, M., Kolasa, M., Skruch, P., Kogut, K., Markiewicz, P., Orlowski, M., Rozewicz, M., Ryszka, A., Sasin, D., Talaska, T., "Design and optimization of hardware-efficient filters for active safety algorithms," *SAE International Journal of Passenger Cars – Electronic and Electrical Systems*, vol. 8, no. 1, 2015, doi: 10.4271/2015-01-0152.

[16] West D.H.D., "Updating Mean and Variance Estimates: An Improved Method", Communications of the ACM, vol.22, 9, p.532-535, 1979